Solutions to J11BA pps 66-85. REV

Solutions that are typeset have been check

Others may or may not have been checke

changed
P68 [2.1]
P 83 [8.3]*
P71 [9.1] [9.2]
P71 [11]*
P73 [4]
P74 [6.1]

P68

✓[1.1] $a_{20} = 4 + 19(3) = 61$

✓[1.2] $a_{25} = 7 + 24(-\frac{1}{3}) = -1$

✓[2.1] $a = 23,\ d = (36-23) = 7 \Rightarrow a_n = 23 + (n-1)7$

✓[2.2] $a = 2,\ d = \frac{5}{4} - 2 = -\frac{3}{4} \Rightarrow a_n = 2 + (n-1)(-\frac{3}{4})$

P69

✓[3] $-37 = 8 + (n-1)(-3) \Rightarrow n = 16 \quad \therefore a_{16}$

✓[4] $\begin{array}{l} a_3 = -4 \\ a_{10} = 38 \end{array} \Rightarrow \begin{bmatrix} a + 9d = 38 \\ a + 2d = -4 \end{bmatrix} \Rightarrow a_n = -16 + (n-1)6$

✓[5] $a = 3,\ a_{15} = 94,$

$3 + 14d = 94 \Rightarrow d = \frac{13}{2}$

$a_5 = 3 + 4(\frac{13}{2}) = 29$

$a_{16} = 3 + 9(\frac{13}{2}) = \frac{123}{2}$

P70

✓[6] Difference of sequential terms is $a_{n+1} - a_n$,

$[p(n+1) + q] - [pn + q] = p$

Since by hypothesis, p constant, $\{pn + q\}$ is arithmetic sequence.

✓[7] $a_n = a + (n-1)d_a$ and $b_n = b + (n-1)d_b$

$a_n + b_n = a + d_a n - d_a + b + d_b n - d_b$

$= (a+b) + d_a n - d_a + d_b n - d_b$

$= (a+b) + d_a(n-1) + d_b(n-1)$

$= (a+b) + (d_a + d_b)(n-1)$

$= \alpha + \beta(n-1)$ where α, β constants.

P71

✓ [8.1] $a=7,\ l=61,\ n=10$

$$S_{10} = \frac{10(7+61)}{2} = 340$$

✓ [8.2] $a=-10,\ d=4,\ n=13$

$$S_{13} = \frac{13[(-20)+(12)4]}{2} = 182$$

[8.3]
changed

$a=21,\ d=-6,\ l=-117$

$$a_n = 21-6(n-1) = -117$$

$$\Rightarrow \quad n = 24$$

$$S_{24} = \frac{24(21-117)}{2} = -1152$$

$$\therefore \quad S_{24} = -1152$$

P71, ctd

[9.1] Prove $1+2+3+\dots+n = \frac{n(n+1)}{2}$

Proof $S_n = 1+2+3+\dots+n$

$S_n = n+(n-1)+(n-2)+\dots [n-(n-1)]$

$2S_n = \underbrace{(n+1)+(n+1)+(n+1)+\dots+n+1}_{n-\text{terms}}$

$\therefore\ S_n = \frac{n(n+1)}{2}$

Alternative Proof

$a=1,\ l=n$

then

$S_n = \frac{n(1+n)}{2}$

$\square$

Use Formula for S_n proved on page 70.

[9.2] Prove $1+3+5+\dots+(2n-1) = n^2$

Proof

$S_n = 1 + 3 + 5 + \dots + (2n-5) + (2n-3) + (2n-1)$

$S_n = (2n-1) + (2n-3) + \dots + 5 + 3 + 1$

$\Rightarrow 2S_n = \underbrace{2n+2n+\dots+2n}_{n \text{ terms}}$

$= (2n)n$

$= 2n^2$

$\therefore\ S_n = n^2$

Alternative Proof

$a=1,\ l=2n-1$

Then $S_n = \frac{n(1+2n-1)}{2}$

$= n^2$

$\square$

✓ [10.1] $S_{100} = \frac{100(101)}{2} = 5050$

✓ [10.2] $S_{66} = 3+6+9+12+\dots+3n$

$= \frac{66(3+198)}{2}$

$S_{66} = 6633$

$n \in \mathbb{Z}^+,\ 3n < 200$

$n < 198$

$\frac{200}{3} = 66\ R\ 2$

so $3 \cdot 66 = 198$

Last mult of 3 before 200

P 71, ctd

[11] $S_n = 297, \; a = 45, \; d = -3 \; ; \; n = ?$

$$297 = \frac{n}{2}\left[90 + (n-1)(-3)\right]$$

$$= \frac{n}{2}\left[90 - 3n + 3\right]$$

$\Rightarrow \; n = 9$ or $n = 22$

$S_9 = 297$ or $S_{22} = 297$

$\therefore \; n = 22, \; n = 9$

END 3.1.1 – 3.1.2

p 73

✓ [1.1] $a_n = a r^{n-1}$

$a_6 = 1 \cdot 2^5$

$a_6 = 32$

✓ [1.2] $a_5 = 3\left(-\frac{1}{3}\right)^4 = 3\left(\frac{1}{81}\right) = \frac{1}{27}$

✓ [2.1] $a = \sqrt{2}$, $r = \sqrt{2}$, $a_n = \sqrt{2}\,(\sqrt{2})^{n-1}$, $a_n = (\sqrt{2})^n$

✓ [2.2] $a = 1$, $r = -3$, $a_n = 1 \cdot (-3)^{n-1}$

✓ [2.3] $a = 1$, $r = (-1)$, $a_n = 1 \cdot (-1)^{n-1}$

✓ [3] $a_n = \dfrac{3^{n+1}}{2^n}$

$a_1 = \dfrac{3^2}{2^1} = \dfrac{9}{2}$

$r = \dfrac{3^{n+2}}{2^{n+1}} \cdot \dfrac{2^n}{3^{n+1}} = \dfrac{3}{2}$

CHANGE

[4]

$a_3 = 4$, $a_5 = 36$

$a_5 = a r^4 = 36$

$a_3 = a r^2 = 4$

$\dfrac{a_5}{a_3} = \dfrac{a r^4}{a r^2} = \dfrac{36}{4}$

$r^2 = 9 \Rightarrow r = 3$ or $r = -3$

then $4 = 3^2 a$ or $4 = (-3)^2 a$

so $a = \frac{4}{9}$

$\therefore$ $a = \frac{4}{9}$, $r = \pm 3$

P 74

✓ [5.1] $a = 3,\ r = 2,\ n = 7$, get S_7

$$S_7 = \frac{3(1-2^n)}{1-2} = -3(1-2^n) = -3 + 3\cdot 2^n$$

$$= -3 + 3\cdot 2^7 = 381$$

✓ [5.2] $a = 1,\ r = -3,\ n = 6$

$$S_6 = \frac{1-(-3)^6}{1+3} = \frac{1}{4}\left(1-(-3)^6\right) = -182$$

✓ [5.3] $a = 5,\ r = \frac{1}{2},\ n = 5$

$$S_5 = \frac{5\left(1-\left(\frac{1}{2}\right)^5\right)}{\frac{1}{2}} = 10\left(1-\frac{1}{32}\right) = 10\cdot\frac{31}{32} = \frac{310}{32} = \frac{155}{16}$$

✓ [6.1] $13, 52, 208, 832, \ldots$

$$r = \frac{52}{13} = 4,\quad a = 13$$

$$S_n = -\frac{13(1-4^n)}{3} = \frac{13[4^n-1]}{3}$$

✓ [6.2] $\sqrt{3}, -1, \frac{1}{\sqrt{3}}, -\frac{1}{3}$

$$r = -\frac{1}{\sqrt{3}},\quad a = \sqrt{3}$$

$$S_n = \frac{\sqrt{3}\left[1-\left(\frac{-\sqrt{3}}{3}\right)^n\right]}{1+\frac{\sqrt{3}}{3}}$$

[7]

Find yearly payment a if

A = amount of loan
r = interest per year
n = total number of years

Let y_k = balance remaining at end of year k.

$y_1 = A(1+r) - a$

$y_2 = [A(1+r) - a]\,(1+r) - a = A(1+r)^2 - a(1+r) - a$

$y_3 = \left[A(1+r)^2 - a(1+r) - a\right](1+r) - a = A(1+r)^3 - a(1+r)^2 - a(1+r) - a$

$\vdots$

$y_n = A(1+r)^n - a(1+r)^{n-1} - a(1+r)^{n-2} - a(1+r)^{n-3} - \ldots - a(1+r)^2 - a(1+r) - a$

The loan is paid off when the balance becomes zero, thus

$A(1+r)^n - a(1+r)^{n-1} - a(1+r)^{n-2} - a(1+r)^{n-3} - \ldots - a(1+r)^2 - a(1+r) - a = 0$

$A(1+r)^n = a + a(1+r) + a(1+r)^2 + a(1+r)^3 + \ldots + a(1+r)^{n-2} + a(1+r)^{n-1}$

The RHS is a geometric series, so

$A(1+r)^n = \frac{a[1-(r+1)^n]}{1-(1-r)}$

Solving for a,

$a = \frac{A\,r\,(r+1)^n}{1-(r+1)^n}$

□